Spectral Estimation Problem-Solving

Methods / Algorithms

Pros & Cons

Testing & Comparing

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Spectral Problem-Solving

Agenda

1. Basic Theory

2. Artifacts: False Peaks

3. Problems:

a. Sinc function do to Windowing (i.e. Limiting time data)

b. Important Peaks, side by side

c. Zero-Fill aka Zero-Padding

4. Power Spectral Density (PSD) Methods vs. True PSD

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SPECTRAL ESTIMATION: BASIC THEORY

The spectral content, $Y(\omega)$, of the process which produces a time dependent (observed) signal, $y(t)$, is mathematically defined as:

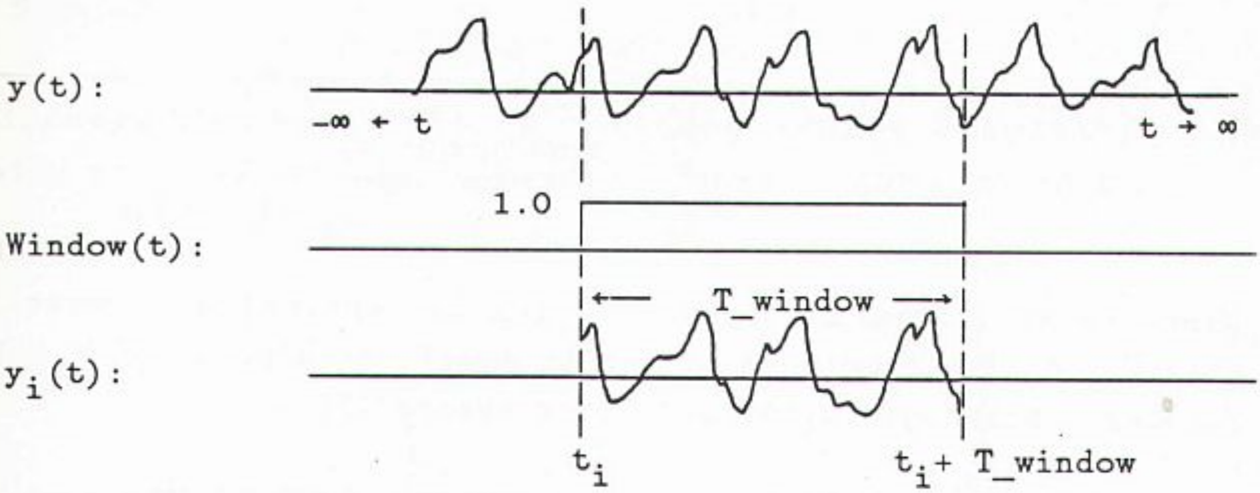
$$Y(\omega) \quad = \quad \int_{-\infty}^{\infty} y(t)\ e^{-j\ \omega\ t}$$

Obviously not all time data, from the beginning to the end of time, is available for calculating the spectral content, $Y(\omega)$. A time window (T_window) segment of $y(t)$ is used for a computer spectral estimation (e.g. Fourier Transform) calculation:

$$y_i(t) \quad = \quad y(t) \cdot \text{Window}(t)$$

$$\text{where:} \quad \text{Window}(t) \quad = \quad \begin{cases} 0 & \text{if } -\infty \le t < t_i \\ 1 & \text{if } t_i \le t \le t_i + \text{T\_window} \\ 0 & \text{if } t_i + \text{T\_window} < t \le \infty \end{cases}$$

For illustration:

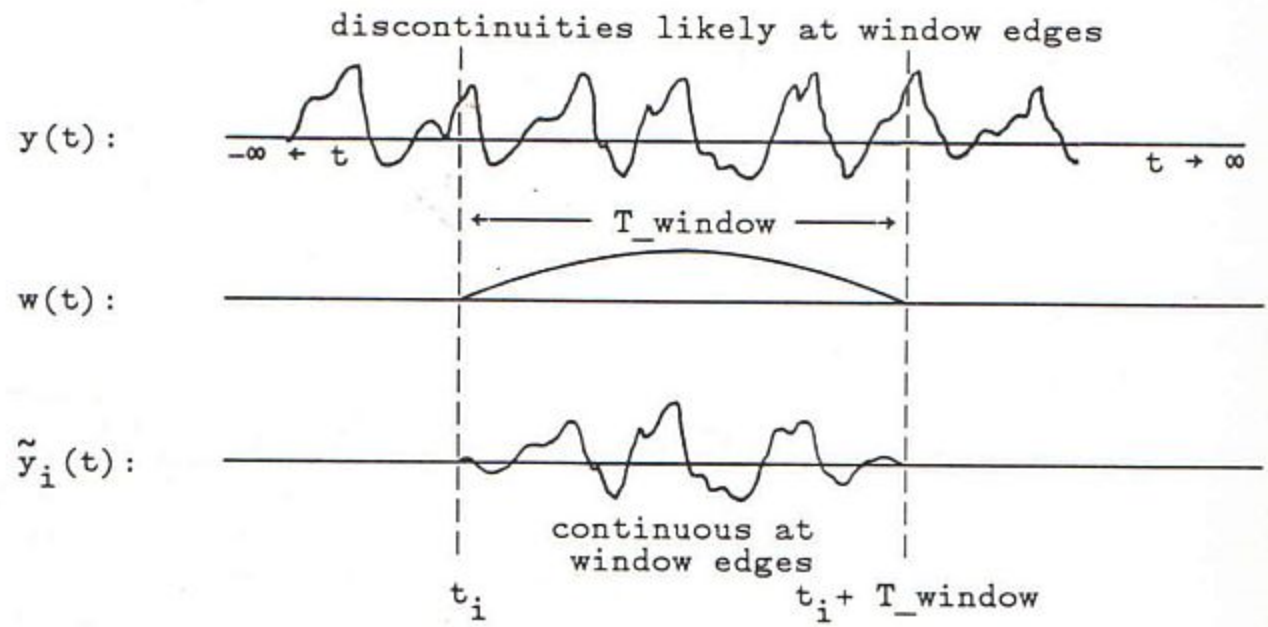


Thus the spectral content would be defined as:

$$Y_i(\omega) \quad = \quad \int_{-\infty}^{\infty} y_i(t)\ e^{-j\ \omega\ t}\,dt \quad = \quad \int_{t_i}^{t_i + \text{T\_window}} y_i(t)\ e^{-j\ \omega\ t}\,dt$$

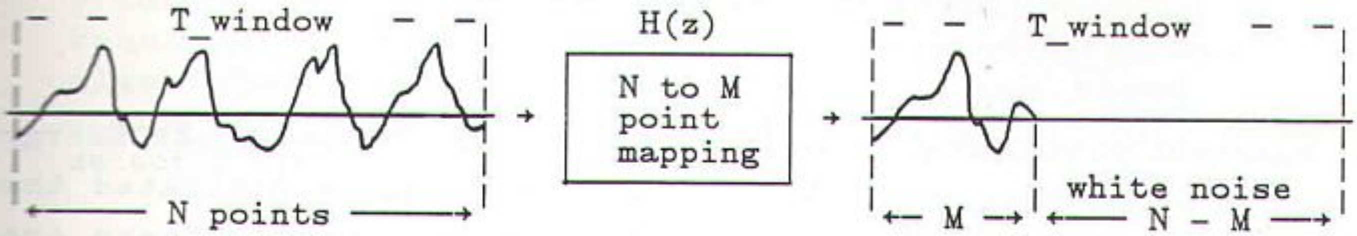
The Fast Fourier Transform (FFT) algorithm was the favorite computer method during the 1970's and 1980's for approximating $Y_i(\omega)$. The FFT assumes $y_i(t)$ is one period of data from the periodic $y(t)$ and that $Y_i(\omega)$ is band-limited. Discontinuities at the beginning and ending of the data window caused $Y_i(\omega)$ to be convolved with a Sinc function. Attempts to remove the Sinc function caused users to investigate the world of windows [3, 4]. The idea was to multiply $y(t)$ by a window function, $w(t)$, that would eliminate the discontinuities (or better yet, the effects thereof) at the window edges. To illustrate:

$$\tilde{y}_i(t) \quad = \quad y(t) \cdot w(t)$$



There is no discontinuity when $\tilde{y}_i(t)$ is repeated and connected to itself. Window functions basically modify each point of $y_i(t)$, thus forcing a bias upon $\tilde{Y}_i(\omega)$ and other errors [5].

Modern spectral estimation algorithms map an input data sequence of N points into a pseudo-sequence process estimate containing only M terms (points). The pseudo-sequence mapping tries to retain the same spectral content as the original N points of data. To do this N - M points are filtered to remove the detected spectrum, leaving no spectral content (i.e. a white noise signal).

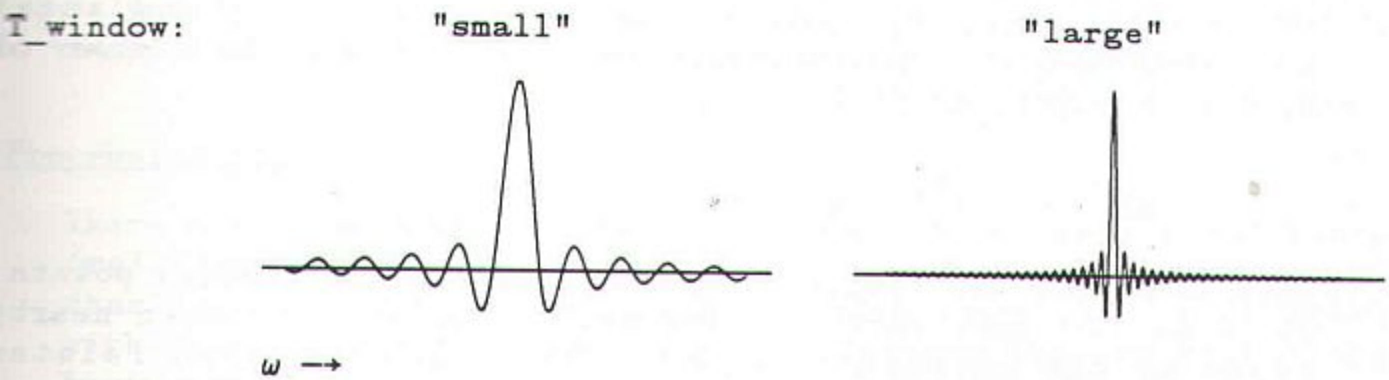


The M points form in effect a shorter "effective T_window" since the N - M points are discarded. This smaller window produces a Sinc function with a lower frequency than if all N points had been used. At this lower frequency the Sinc function sidelobe peaks are farther apart and thus there are fewer of them within the Nyquist frequency limit. This reduces the number of false peaks (i.e. spectral artifacts) which aids in determining the true spectral content, $Y(\omega)$. ("M" is defined as the sum of the parameters NPOLES and NZEROS.)

In order to illustrate the affect on a Sinc function of varying the data window length, note that the Sinc function has the form:

$$\mathrm{Sinc}(\omega) = \frac{\mathrm{Sin}(\omega/\text{effective T\_window})}{\omega/\text{effective T\_window}}$$

Thus increasing the effective T_window length results in the following relative changes in the Sinc function for a given ω.



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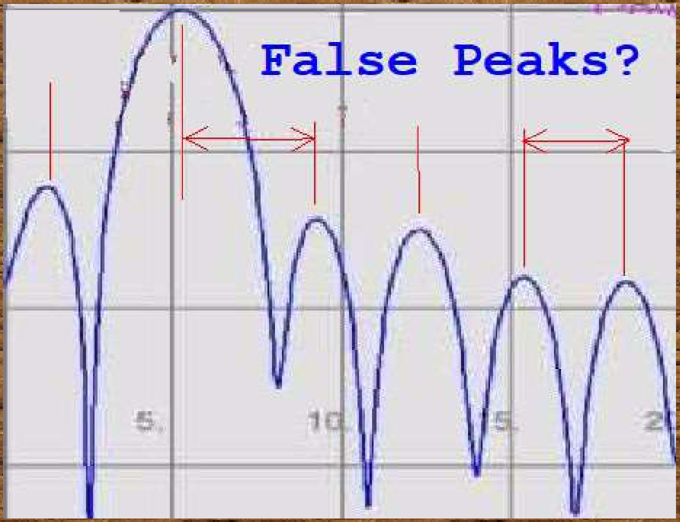
Artifacts: False Peaks



If distance between one or more peaks is the same, you may have detected a false peak. Often false peaks are found next to a main peak and its adjacent sidelobe peaks as shown in above waveform.

Another thing to watch for is when adjacent peak amplitudes decay.

Integration Process for Fourier Transform (FT)

- Sample size (i.e. nPoints)
- Effects from Windowing data

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Sample size

$$df = \frac{1}{(nPoints - 1) * dt}$$

Smaller 'df' implies

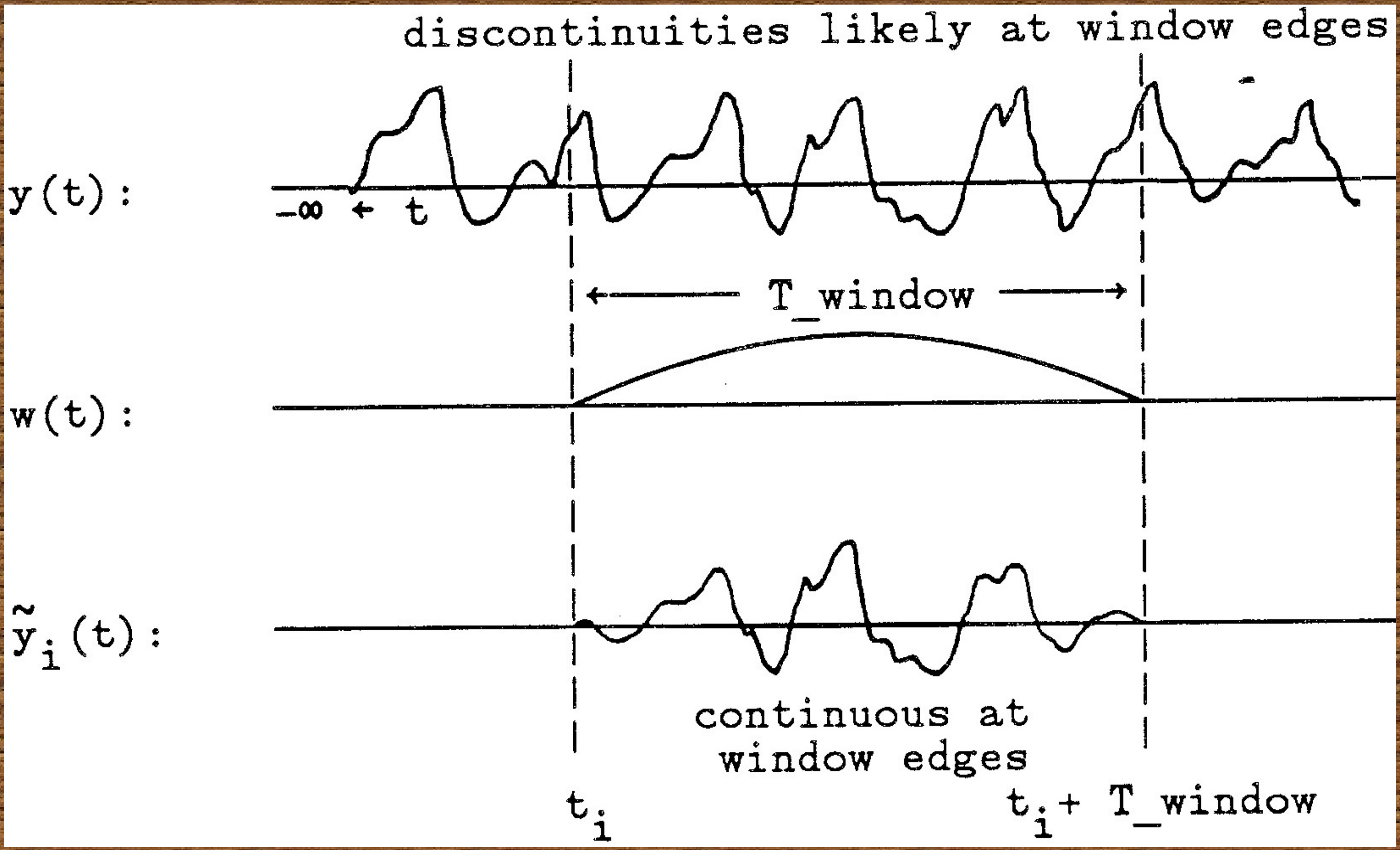
- *more 'nPoints'*

  and/or

- *larger 'dt'*

Smaller 'df' provides better spectral resolution!

Windowing Data



Integration

$$\int_{-\infty}^{\infty} = \int_{-\infty}^{t1} + \int_{t1}^{t1+t\_window} + \int_{t1+t\_window}^{\infty}$$

$$\int_{-\infty}^{\infty} = \int_{t1}^{t1+t\_window}$$

Multiplication implies Convolution

Multiple in one domain
implies
Convolution in other domain

____

f x g ==> F * G

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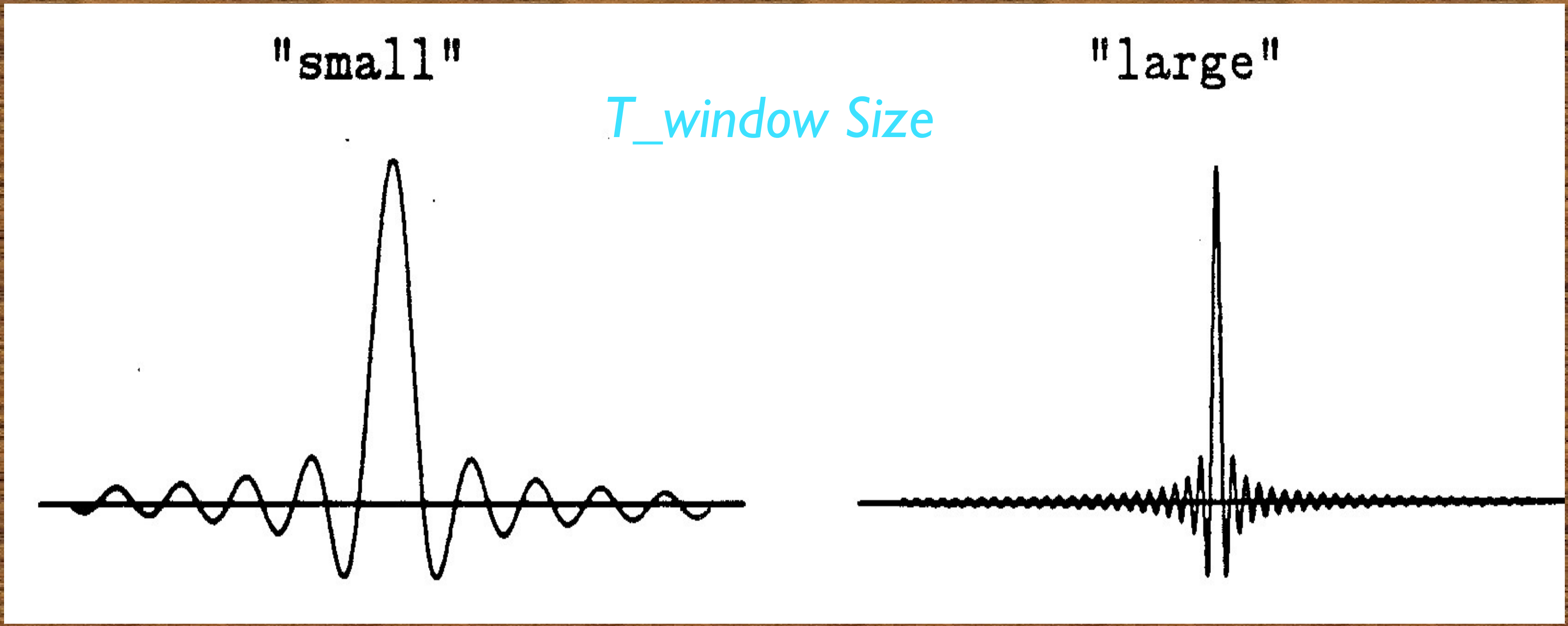
Spectral Problem-Solving

Spectral Est. Problems

- *Sinc function do to Windowing (i.e. Limiting time data)*
- *Important Peaks, side by side*
- *Zero-Fill aka Zero-Padding*

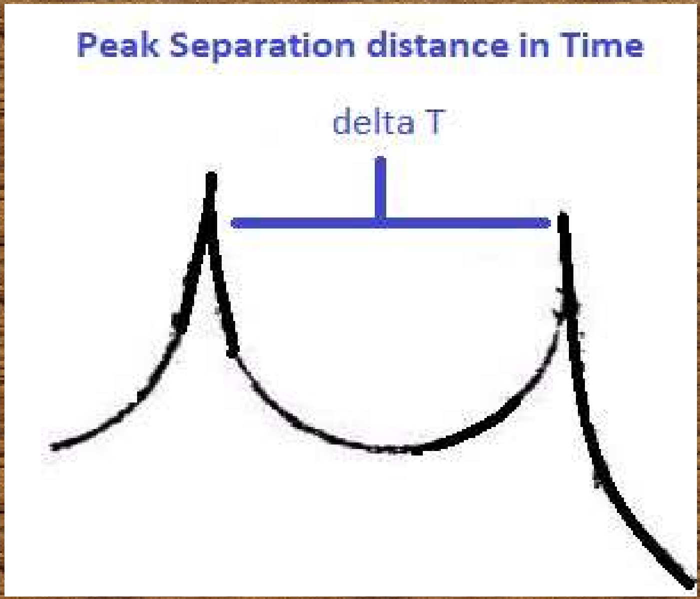
Windowing Artifacts

T_window may produce a Sinc function that is convolved with your true PSD



Spectral Problem-Solving

Important Peaks, side by side



As peaks get closer to each other, determining both peaks becomes a challenge for various PSD methods. Kind of a give & take. Give/allow some Artifacts in order to take/get more Spectral content.

Smaller 'delta T', fewer peaks detected by many methods.

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Zero-Fill Effects

Most Fourier Transfer (FT) and Power Spectral Density (PSD) methods use arrays to estimate a Spectrum content. Given 'nPoints' of time-data a method's arrays must be larger than nPoints. The extra add-on cells are called Zero-Fill since their value are Zero. The following plots show the same Spectrum, but continue to fill-in the spectrum as the array size increases. Are your arrays large enough for your project?

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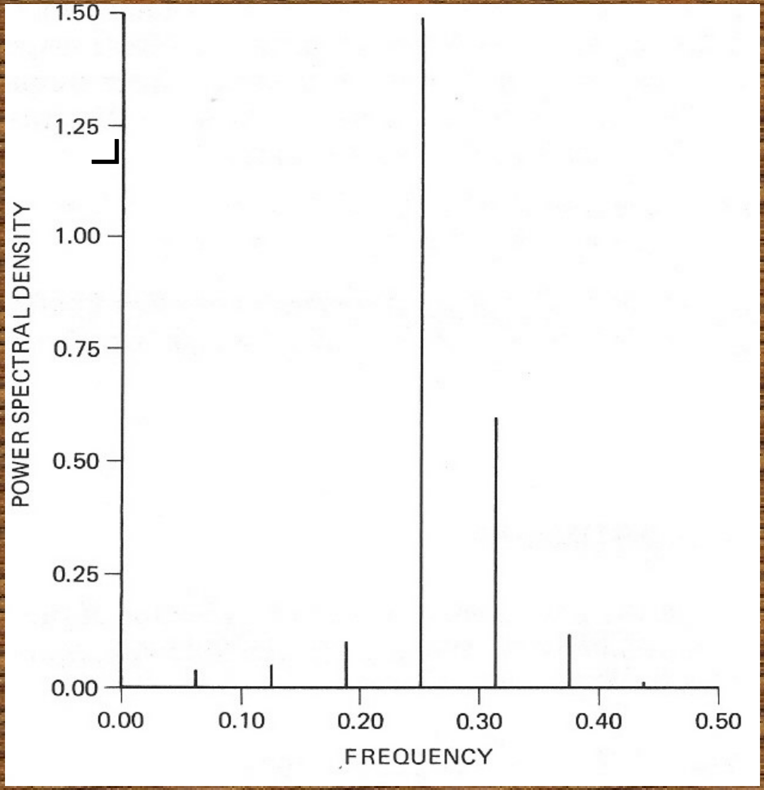
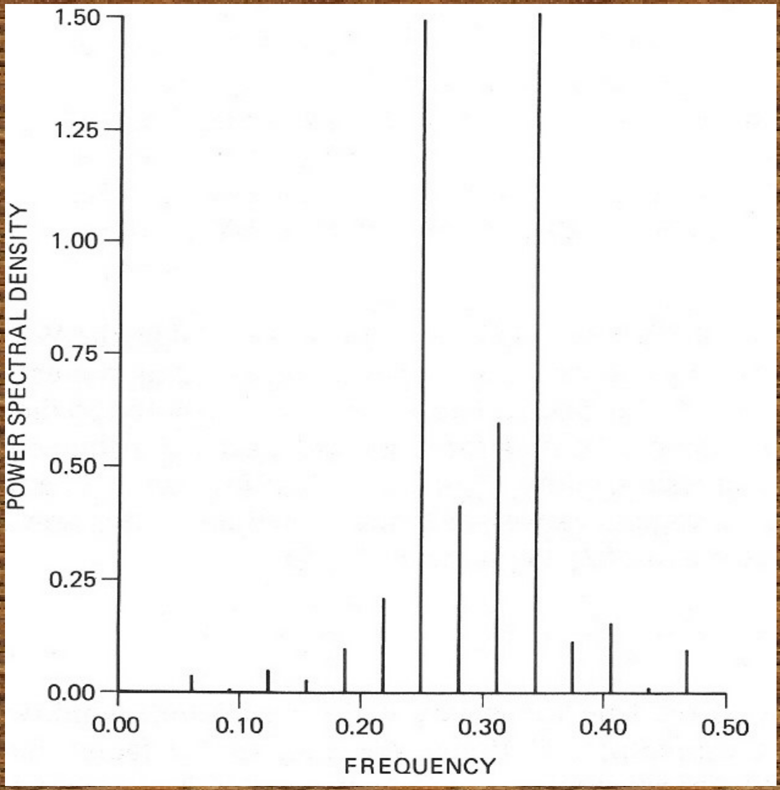
Zero-Padding or Zero-Filling

nPoints

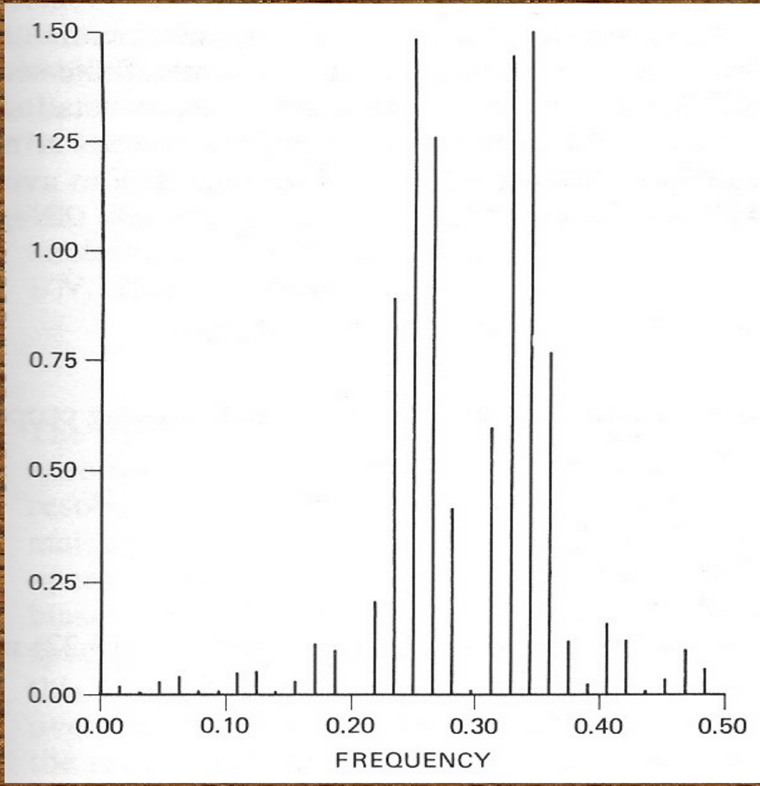


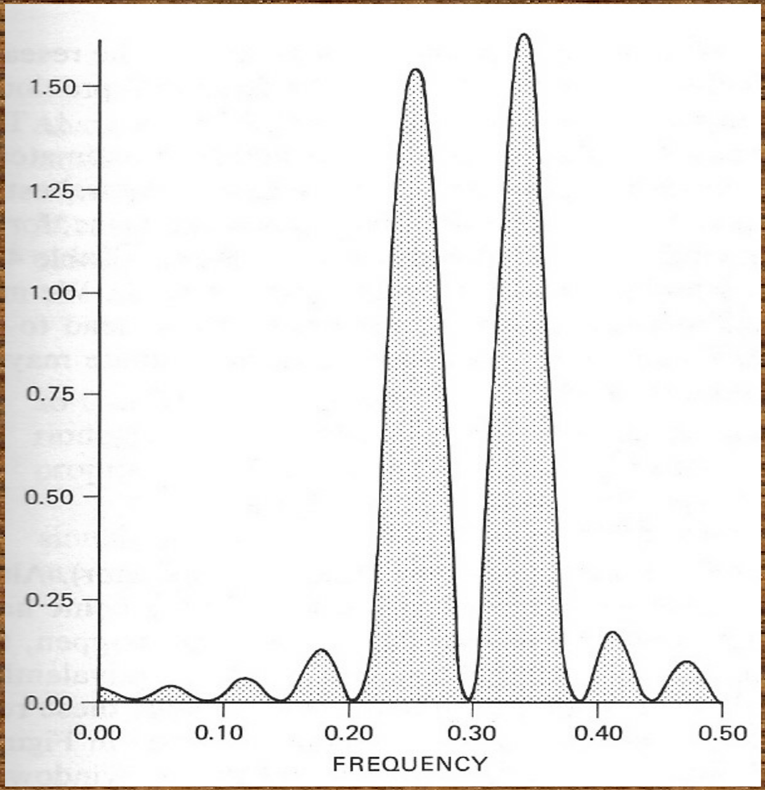
2 * nPoints



Zero-Padding or Zero-Filling (cont.)

4 * nPoints ... 32 * nPoints





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Power Spectral Density (PSD) Methods vs. True PSD

The following plots show different responses to various methods used to estimate the True PSD shown in the center plot. The Time-data used for all these responses is attached for your testing of other methods. How does your method compare to the True PSD plot? This test should give an idea where your methods are strong or weak.

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Time Dataset for. True PSD

.01	(6.3307, -0.174915)
.02	(-1.33539, -0.03044)
.03	(3.61896, -0.260459)
.04	(1.87513, -0.323974)
.05	(-1.08561, -0.136055)
.06	(3.99114, -0.101864)
.07	(-4.10184, 0.130571)
.08	(1.55399, 0.0977916)
.09	(-2.1258, -0.306485)
.10	(-3.27873, -0.0544436)
.11	(0.241218, 0.0962379)
.12	(-5.74708, 0.0186908)
.13	(-0.0165977, 0.237493)
.14	(-3.28921, -0.188478)
.15	(-1.31227, -0.120636)
.16	(0.745251,-0.0679575)

'Y' values are complex numbers

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.17	(-1.77199, -0.416229)
.18	(2.56419, -0.270373)
.19	(0.21325, -0.232544)
.20	(2.23409, 0.236383)
.21	(2.2949, 0.173061)
.22	(1.09186, 0.140938)
.23	(2.29353, 0.442044)
.24	(0.695823, 0.509325)
.25	(0.759858, 0.417967)
.26	(-0.354267, 0.506891)
.27	(-0.594517, 0.39708)
.28	(-1.88618, 0.649179)
.29	(-1.39041, 0.867086)
.30	(-3.06381, 0.422965)
.31	(-2.0433, 0.0825514)
.32	(-2.1628, -0.0933218)

Download a free App for testing 10+ other PSD methods at
goal-driven.net/apps/spectrumsolvers.html

True PSD for Comparison

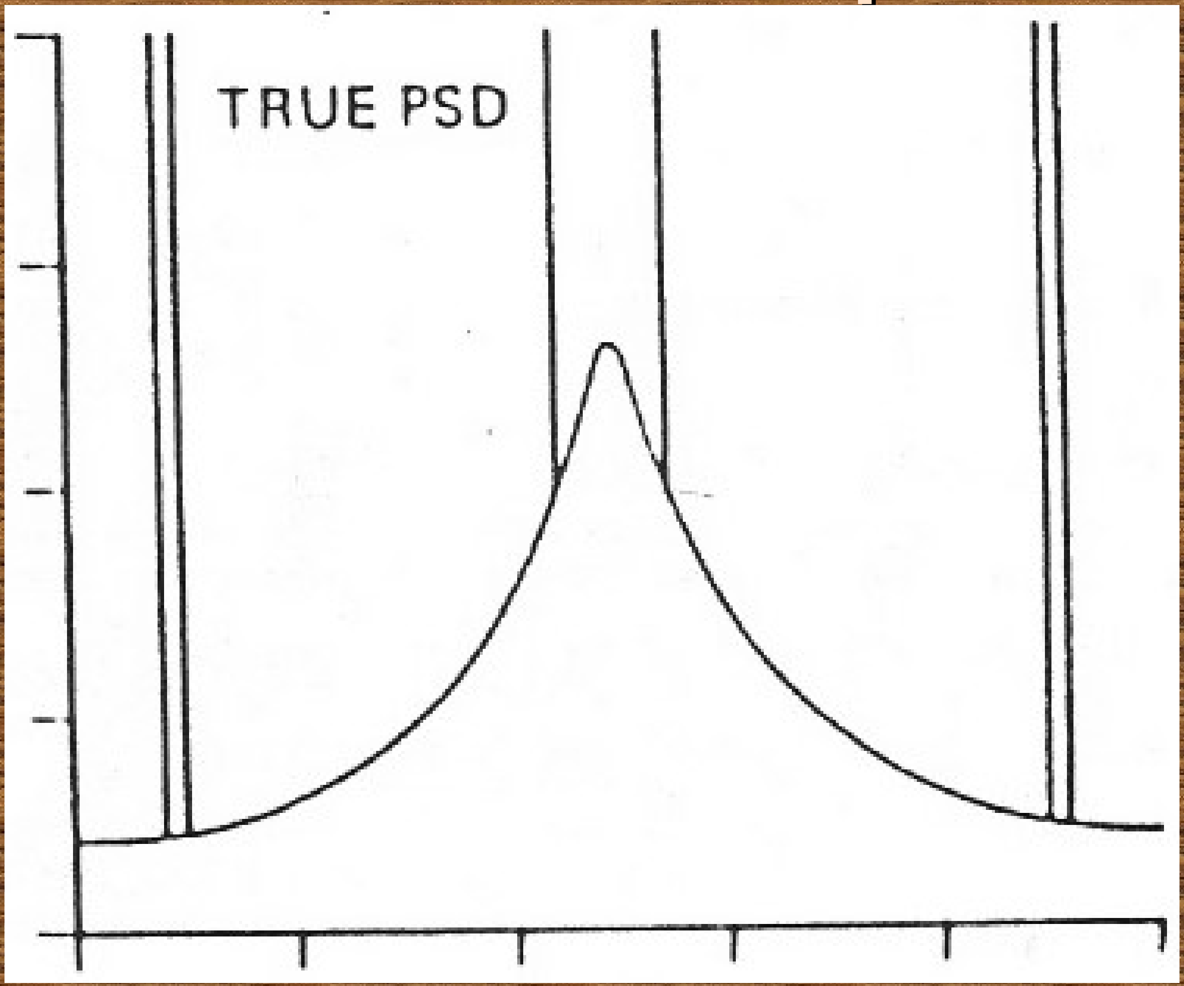

ILLUSTRATION OF SPECTRAL ESTIMATION METHODS

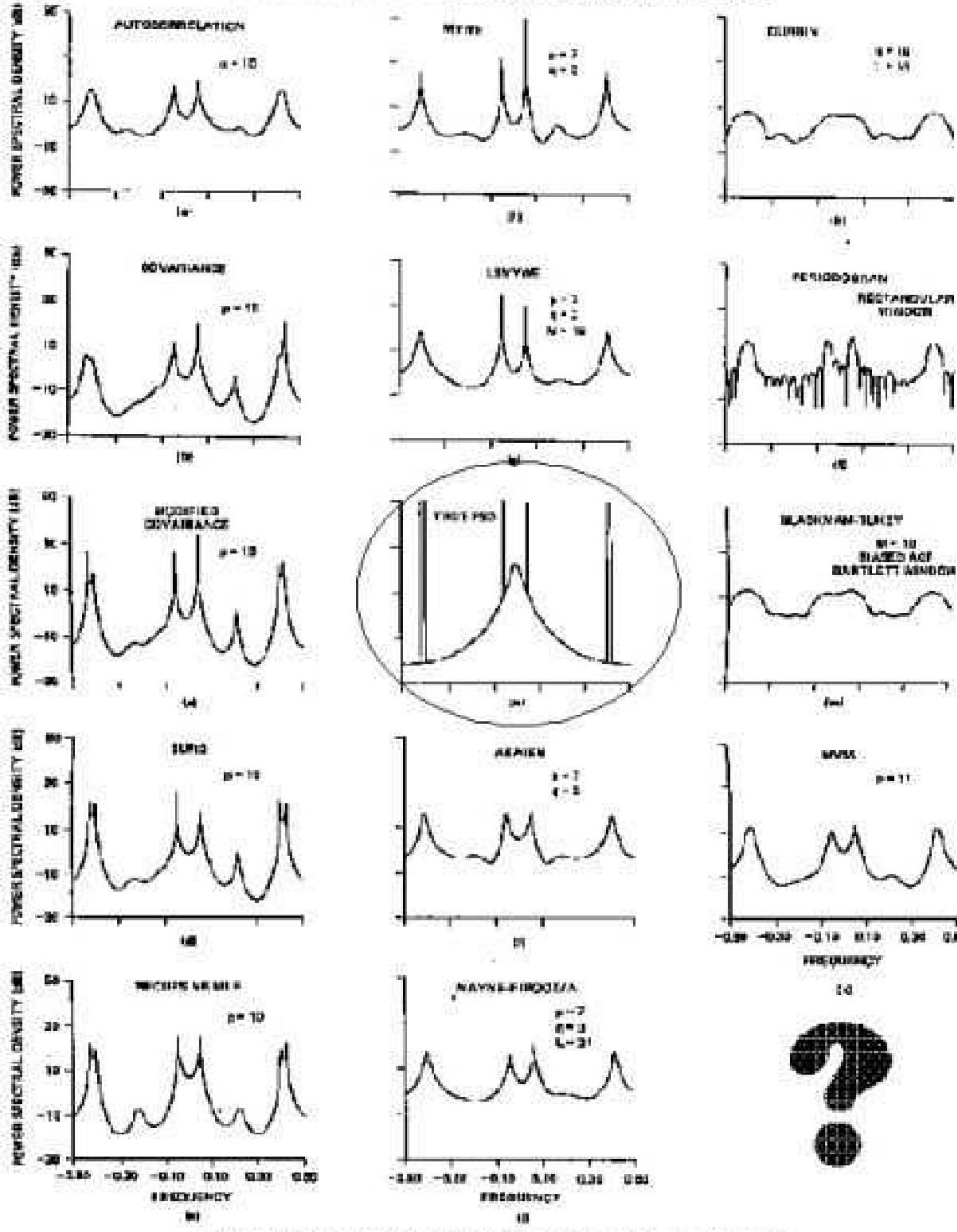

Figure 12.1 Summary of estimator results. (a) Autocorrelation, (b) Covariance, (c) Modified covariance, (d) Burg, (e) Recursive MLE, (f) MYWE, (g) LSMYWE, (h) True PSD, (i) Akaike, (j) Mayne–Firoozan, (k) Durbin, (l) Periodogram, (m) Blackman–Tukey, (n) MYSE.

Kay, S.M., "Modern Spectral Estimation: Theory and Applications", Prentice-Hall, Englewood Cliffs, NJ, pp 398-399, 1988.

Spectral Problem-Solving

Summary

- Artifacts: eliminated all of them? In the process, does your FT/PSD method detect all spectrum above a minimum threshold?
- Zero-Fill: enough padding to give good frequency resolution?
- Peak-Density: able to 'see' side by side peaks of importance?

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